

Study Guide and Intervention**Rational Exponents****Rational Exponents and Radicals**

Definition of $b^{\frac{1}{n}}$	For any real number b and any positive integer n , $b^{\frac{1}{n}} = \sqrt[n]{b}$, except when $b < 0$ and n is even.
Definition of $b^{\frac{m}{n}}$	For any nonzero real number b , and any integers m and n , with $n > 1$, $b^{\frac{m}{n}} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$, except when $b < 0$ and n is even.

Example 1 Write $28^{\frac{1}{2}}$ in radical form.Notice that $28 > 0$.

$$\begin{aligned}
 28^{\frac{1}{2}} &= \sqrt{28} \\
 &= \sqrt{2^2 \cdot 7} \\
 &= \sqrt{2^2} \cdot \sqrt{7} \\
 &= 2\sqrt{7}
 \end{aligned}$$

Example 2 Evaluate $\left(\frac{-8}{-125}\right)^{\frac{1}{3}}$.Notice that $-8 < 0$, $-125 < 0$, and 3 is odd.

$$\begin{aligned}
 \left(\frac{-8}{-125}\right)^{\frac{1}{3}} &= \frac{\sqrt[3]{-8}}{\sqrt[3]{-125}} \\
 &= \frac{-2}{-5} \\
 &= \frac{2}{5}
 \end{aligned}$$

Exercises

Write each expression in radical form.

1. $11^{\frac{1}{7}}$

2. $15^{\frac{1}{3}}$

3. $300^{\frac{3}{2}}$

Write each radical using rational exponents.

4. $\sqrt{47}$

5. $\sqrt[3]{3a^5b^2}$

6. $\sqrt[4]{162p^5}$

Evaluate each expression.

7. $-27^{\frac{2}{3}}$

8. $\frac{5^{-\frac{1}{2}}}{2\sqrt{5}}$

9. $(0.0004)^{\frac{1}{2}}$

10. $8^{\frac{2}{3}} \cdot 4^{\frac{3}{2}}$

11. $\frac{144^{-\frac{1}{2}}}{27^{-\frac{1}{3}}}$

12. $\frac{16^{-\frac{1}{4}}}{(0.25)^{\frac{1}{2}}}$

5-7 Study Guide and Intervention (continued)**Rational Exponents**

Simplify Expressions All the properties of powers from Lesson 5-1 apply to rational exponents. When you simplify expressions with rational exponents, leave the exponent in rational form, and write the expression with all positive exponents. Any exponents in the denominator must be positive integers.

When you simplify radical expressions, you may use rational exponents to simplify, but your answer should be in radical form. Use the smallest index possible.

Example 1 Simplify $y^{\frac{2}{3}} \cdot y^{\frac{3}{8}}$.

$$y^{\frac{2}{3}} \cdot y^{\frac{3}{8}} = y^{\frac{2}{3} + \frac{3}{8}} = y^{\frac{25}{24}}$$

Example 2 Simplify $\sqrt[4]{144x^6}$.

$$\begin{aligned}\sqrt[4]{144x^6} &= (144x^6)^{\frac{1}{4}} \\ &= (2^4 \cdot 3^2 \cdot x^6)^{\frac{1}{4}} \\ &= (2^4)^{\frac{1}{4}} \cdot (3^2)^{\frac{1}{4}} \cdot (x^6)^{\frac{1}{4}} \\ &= 2 \cdot 3^{\frac{1}{2}} \cdot x^{\frac{3}{2}} = 2x \cdot (3x)^{\frac{1}{2}} = 2x\sqrt{3x}\end{aligned}$$

Exercises

Simplify each expression.

1. $x^{\frac{4}{5}} \cdot x^{\frac{6}{5}}$

2. $(y^{\frac{2}{3}})^{\frac{3}{4}}$

3. $p^{\frac{4}{5}} \cdot p^{\frac{7}{10}}$

4. $(m^{-\frac{6}{5}})^{\frac{2}{5}}$

5. $x^{-\frac{3}{8}} \cdot x^{\frac{4}{3}}$

6. $(s^{-\frac{1}{6}})^{-\frac{4}{3}}$

7. $\frac{p}{p^{\frac{1}{3}}}$

8. $(a^{\frac{2}{3}})^{\frac{6}{5}} \cdot (a^{\frac{2}{5}})^3$

9. $\frac{x^{-\frac{1}{2}}}{x^{-\frac{1}{3}}}$

10. $\sqrt[6]{128}$

11. $\sqrt[4]{49}$

12. $\sqrt[5]{288}$

13. $\sqrt{32} \cdot 3\sqrt{16}$

14. $\sqrt[3]{25} \cdot \sqrt{125}$

15. $\sqrt[6]{16}$

16. $\frac{x - \sqrt[3]{3}}{\sqrt{12}}$

17. $\sqrt{\sqrt[3]{48}}$

18. $\frac{a\sqrt[3]{b^4}}{\sqrt{ab^3}}$